DYNACOMP

HARMONIC ANALYZER

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INTRODUCTION

HARMONIC ANALYZER is a frequency spectrum analysis package designed with the engineer in mind. It applies the concept of the Fast Fourier Transform (FFT) to an input data set to provide a frequency domain representation of the function approximated by that input data. HARMONIC ANALYZER is based on DYNACOMP's popular FOURIER ANALYZER, but includes special data handling features.

The user may save and recall both the input data file and previously calculated results. Data files may be loaded (from cassette, diskette, or disk), added to, deleted from, and generally edited. As with FOURIER ANALYZER, the input data may be plotted before the calculations are started. The frequency domain results may also be plotted as well as saved (to be later recalled for new plots). The analyses are easily repeated without re-entering the data. This is handy when you wish to test the sensitivity of the results to a particular data point.

One of the unique features of HARMONIC ANALYZER is that the data need not be equally spaced in "time." A special spline interpolation is performed just prior to the FFT calculation to convert the the unequally spaced data into a new "data" file which is uniformly spaced as required by the FFT.

RELATION TO OTHER DYNACOMP FREQUENCY TRANSFORM PROGRAMS

DYNACOMP's FOURIER ANALYZER and TRANSFER FUNCTION ANALYZER were designed to be used on limited duration signals. The resulting transforms, therefore, represent a continuous spectrum. HARMONIC ANALYZER is meant to handle another class of signals; those which are repetitive and which, therefore, have discontinuous spectra. If FOURIER ANALYZER or TFA are used on single period waveforms belonging to repetitive wave-trains, the results will be wrong (not the fault of the program). Similarly, if HARMONIC ANALYZER is applied to limited duration signals, the results are very misleading. The truncated sinewave example given later demonstrates this.

THEORY

Repetitive signals may be decomposed into sinusoidal components as follows:

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{iw_n t}$$

where

$i = \sqrt{-1}$

$W_n = nW_o$ (radians/second)

$W_o = 2\pi/T_o$ (radians/second)

T_o = period of signal (seconds)

The Fourier Transform is defined as:

$$F(W) = \int_{-\infty}^{\infty} f(t)e^{-iwt}\, dt$$

Inserting the series representing f(t), we get:

$F(W) = a_n$ $(W=W_n)$

$= 0$ (otherwise)

The Fourier Transform of repetitive signals is, therefore, discontinuous; composed of spikes.

As a theoretical example, consider the following function:

$f(t) = \frac{1}{2}$ $-1 \leq t \leq 1$

$f(t) = 0$ elsewhere

This is not a repetitive waveform.

The associated Fourier Transform is a "sinc" function:

$$F(w) = \frac{\sin(w)}{w}$$

Note that this is a continuous function.

If instead the function were:

$$F(t) = \tfrac{1}{2} \qquad -1 \leq t \leq 1$$

repeating at $t=0$, $t=nT_o$

where $T_o \geq 2$

The spectrum would be discrete:

$$F_n = F(W_n) = \frac{\sin(W_n)}{W_n}$$

The larger T_o is, the closer together will be the W_n, and in the limit as $T_o \to \infty$ the spectrum will become continuous.

A classical example of the difference between the Fourier integral for limited duration signals and the DFT/FFT is the spectrum derived for a limited duration sinewave. The DFT and FFT algorithms generally give a spectrum with a simple spike at the sinewave frequency. This is what one would expect for a continuous (infinite in extent) sinewave. The Fourier integral instead presents a spectrum in which the signal energy is concentrated about the sinewave frequency, but with some spread (actually an offset "sinc" function).

APPROXIMATION ERROR

By using only a finite number of data points to represent a (piece-wise) continuous function, we are introducing approximation error. In effect, the function is being digitized*, or sampled. Thus, it is not possible to accurately and unambiguously capture the high frequency components of the original function. This limitation is described by the Nyquist sampling theorem which, in essence, states that the sampling rate (number of samples/unit time) should be at least twice the highest frequency component contained in the signal. For sampling rates below this, "aliasing" can occur. Spectrum calculations beyond this point should be considered suspect, though not necessarily invalid. Usually one avoids this problem by "filtering" the data before sampling. This may be done using the DATA SMOOTHER program also available through DYNACOMP, or DIGITAL FILTER (available February 1981). If the data input to HARMONIC ANALYZER is equally spaced, the sampling rate will be twice the highest frequency calculated, though perhaps not at or greater than twice the rate of the highest component in the original signal.

THE NUMBER OF DATA POINTS

The FFT algorithm employed in HARMONIC ANALYZER requires that the number of data points processed be equal to an even power of 2 (e.g., 2, 4, 8, 16, 32, etc.). You will not be permitted to enter the transform step unless that is the case. The data file structure used by HARMONIC ANALYZER is generally the same as that employed by TFA (Transfer Function Analyzer). Thus, it is possible to load a data set which violates this condition, but data will have to be added or deleted to bring the number of points to an even power of two.

NON-UNIFORM, DISORDERED DATA

Before HARMONIC ANALYZER enters the transform stage, a new data set is constructed from the one supplied. First, the "old" data set is ordered in terms of ascending "X" (or "t"). Next, the range of the N data points is determined and a new set of N equally spaced "X" (or "t") values chosen. Using a cubic spline interpolation, the "Y" (of "f") values corresponding to these equally spaced,

---

* Actually, digitization is a further source of error.

ordered values are then determined. This artificial data set is subsequently sent to the FFT subroutine. If the original data set was ordered and equally spaced, the artificial data set is just an image of the original data collection.

A cubic spline interpolation scheme was chosen over a cubic LaGrange (or Newton) in order to avoid the "ringing" associated with the latter.

USING HARMONIC ANALYZER TO CALCULATE FREQUENCY RESPONSE

HARMONIC ANALYZER may be directly applied to analyze the transfer function of a linear system. The experimental component is simple. A repetitive string of delta functions (e.g., pulse train of "spikes") is used as the input to the system to be analyzed. If the spike is transmitted unaltered, then $F(w_n)$ = a constant. The Fourier transform of the delta function is unity. Therefore, if the spike is transmitted with distortion, the transform (as calculated by HARMONIC ANALYZER) of this distorted waveform is proportional to the frequency response of the system being tested. An example is given in the attached listings.

A word of caution is in order. Be careful that the input pulse is not so large that non-linear distortion occurs, or too wide. Ideally, the width should be less than the distance between sampling points.

PLOTTING WIDTH

The aspect ratio of the plotting routine is controlled by the parameter U. By increasing the value of U, fewer line feeds are inserted between plotted points. U is defined at the very beginning of the program and may be adjusted by the user to suit the terminal device employed.

In conclusion, HARMONIC ANALYZER is a fairly general and very powerful frequency analysis tool. To get the best results from its application, remember that it is a numerical approximation whose accuracy and resolution improves with more precise and extensive data.

HARMONIC ANALYZER EXAMPLES

The following set of examples was generated using the North Star version of HARMONIC ANALYZER. All other versions are very similar in their behavior.

```
*** HARMONIC ANALYZER ***


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     DYNACOMP
     P.O. BOX 162
     WEBSTER, NEW YORK  14580


THIS PROGRAM TRANSFORMS A GIVEN
REPETITIVE DATA SET INTO ITS
FREQUENCY DOMAIN REPRESENTATION
USING THE FAST FOURIER TRANSFORM.


THE USER INPUTS THE NUMBER
OF DATA POINTS TO BE TREATED.
THIS NUMBER MUST BE AN EVEN POWER
OF TWO. FOR EXAMPLE: N=2^K.                    E.G., 2, 4, 8, 16, 32
THE PROGRAM WILL THEN PLOT
THE DATA AND ITS SPECTRUM IN
TERMS OF AMPLITUDE AND PHASE.                  The phase is restricted to the range 0-360°

YOU MAY ALSO CHOOSE TO RECALL
PREVIOUS DATA OR RESULTS.

DO YOU WISH TO PLOT
PREVIOUS RESULTS (Y/N)? ?N                     A "Y" answer will lead to prompts concerning
                                               loading a previously saved "results" file.
                                               Control will then go directly to the plotting
                                               routines for amplitude and phase.

INPUT NUMBER OF TABLE VALUES? ?16              You will not be able to pass this point with
                                               an illegal number of data pairs.

IS THE DATA TO BE LOADED
FROM DISK (Y/N)? ?N                            If the data were to be loaded from disk (or
                                               cassette), the number of table values chosen
                                               above must be larger than the number of pairs
INPUT THE DATA. YOU MAY EDIT IT LATER.         in the file to be loaded. In this case we will
                                               build a 16 point data table and save it later.
X( 1) = ?1
                Y( 1) = ?0
X( 2) = ?2
                Y( 2) = ?0
X( 3) = ?3                                     At this stage data is entered until 16 pairs are
                Y( 3) = ?0                     recorded. If a mistake is made it can be corrected
X( 4) = ?4                                     later. Note that the data need not be entered in
                Y( 4) = ?0                     "time" order. It will be sorted later.
X( 5) = ?5
                Y( 5) = ?0
X( 6) = ?6
                Y( 6) = ?1
X( 7) = ?7
                Y( 7) = ?1
X( 8) = ?8                                     The example we will first examine is a simple
                Y( 8) = ?1                     train of broad pulses.
X( 9) = ?9
                Y( 9) = ?1
X( 10) = ?10
                Y( 10) = ?1
X( 11) = ?11
                Y( 11) = ?1
X( 12) = ?14
                Y( 12) = ?0
X( 13) = ?15
                Y( 13) = ?0
X( 14) = ?16
                Y( 14) = ?0
X( 15) = ?17                                   This is an intentional error to be corrected.
                Y( 15) = ?0
X( 16) = ?12                                   Observe that this data point is out of order.
                Y( 16) = ?0
```

```
0
I----I----I----I----I----I----I
*                             I  1       -.375
:                             I
*                             I  2       -.375
:                             I
*                             I  3       -.375
:                             I
*                             I  4       -.375
:                             I
*                             I  5       -.375
:                             I
:                        *    I  6       .625
:                             I
:                        *    I  7       .625
:                             I
:                        *    I  8       .625
:                             I
:                        *    I  9       .625
:                             I
:                        *    I  10      .625
:                             I
:                        *    I  11      .625
:                             I
*                             I  12      -.375
:                             I
*                             I  13      -.375
:                             I
*                             I  14      -.375
:                             I
*                             I  15      -.375
:                             I
*                             I  16      -.375
I----I----I----I----I----I----I
CONTINUE?Y
```

The ordered data set is plotted without modification. The values printed to the right are the result of removing the DC component. The printed values are the ones sent to the interpolation routine.

Always try to use a plot width value greater than the number of data pairs. otherwise plot values may have to double-up. If more than one point is plotted on a given line, it is not possible to also display the values to the right.

A maximum of two plot values will be shown on a given plot line. The third and higher will be ignored.

```
FOURIER TRANSFORM UNDER WAY....

 FREQUENCY    AMPLITUDE    PHASE
CYCLES/UNIT    SCALED     DEGREES
-----------  ----------  -------

     0           0          0
     .0625       1          191
     .125        .3902      22
     .1875       .1455      34
     .25         .2986      225
     .3125       .0972      56
     .375        .1616      67
     .4375       .1989      259
     .5          0          0

DO YOU WISH TO EDIT ANY DATA PAIR (Y/N)? ?Y

WHICH PAIR? ?15

X( 15) = ?13
          Y( 15) = ?0

DO YOU WISH TO EDIT ANY DATA PAIR (Y/N)? ?N

SORTING DATA...


DO YOU WISH TO SAVE THE CURRENT
DATA ON DISK (Y/N)? ?N

DO YOU WISH TO REMOVE THE
DC COMPONENT (Y/N)? ?Y

PLOT DATA (Y/N)? ?Y

WHAT IS THE PLOT WIDTH? ?30

DISPLAY PLOT VALUES (Y/N)? ?Y
```

There is somewhat of a pause before the values are displayed.

Observe that if the number of data pairs is N, the number of transform pairs is N/2+1.

Data may be corrected by changing it as shown to the left.

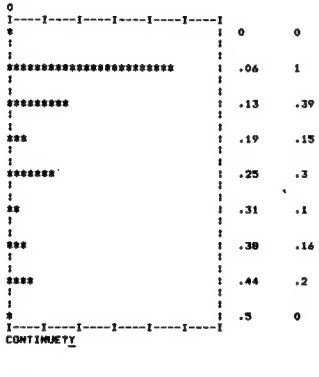
First the data is sorted to put it in ascending X-order. Then interpolation is performed to ensure that the data (as seen by the FFT) is equal-spaced in X. This is a slow step.
If we chose to save the data on disk (or cassette), we would be prompted for a file name. In the cassette versions a name is requested, but actually not used.

In most versions this refers to the number of horizontal print positions.
You may opt to have the data pair values printed to the right of the plot (not available for Atari).

```
DO YOU WISH AN AMPLITUDE
VERSUS FREQUENCY PLOT (Y/N)! ?Y

WHAT IS THE PLOT WIDTH! ?30

DISPLAY PLOT VALUES (Y/N)! ?Y
```



This plot is actually a discrete form of the "sinc" function, but missing the DC term since we chose to remove it from the data to start with. A bar graph pattern is employed to remind you that that the transform is discontinous.

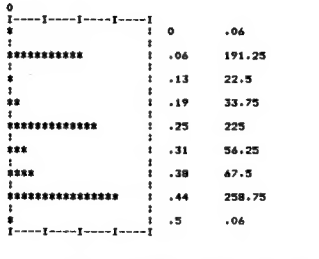
```
CONTINUEY
```

The calculation and plot ends at the Nyquist frequency associated with the span of the data ($X_N - X_1$; ordered) and the number of data points. It is the frequency associated with the interpolated sampling frequency.

```
DO YOU WISH A PHASE PLOT (Y/N)! ?Y

WHAT IS THE PLOT WIDTH! ?20

DISPLAY PLOT VALUES (Y/N)! ?Y
```




```
DO YOU WISH TO SAVE THESE RESULTS (Y/N)! ?N

DO YOU WISH TO REPEAT THE ANALYSIS
USING THE ORIGINAL DATA (Y/N)! ?Y
```

```
CURRENT DATA FILE:

 I     X(I)       Y(I)
---   ------     ------

 1     1          0
 2     2          0
 3     3          0
 4     4          0
 5     5          0
 6     6          1
 7     7          1
 8     8          1
 9     9          1
 10    10         1
CONTINUE?Y

 11    11         1
 12    12         0
 13    13         0
 14    14         0
 15    15         0
 16    16         0

DO YOU WISH TO ADD TO DATA (Y/N)? ?Y

SORRY. ARRAY SPACE FILLED.

DO YOU WISH TO DELETE DATA (Y/N)? ?Y

SUGGESTION - DELETE STARTING AT THE
END OF THE FILE.

WHICH DATA PAIR? ?16

DO YOU WISH TO DELETE DATA (Y/N)? ?N

ERROR! THE CURRENT NUMBER OF DATA
PAIRS IS NOT AN EVEN POWER OF 2.

DO YOU WISH TO ADD TO DATA (Y/N)? ?Y

YOU MAY ADD  1 POINTS.

HOW MAY POINTS DO YOU WISH TO ADD? ?1

X( 16) = ?16
Y( 16) = ?0

DO YOU WISH TO DELETE DATA (Y/N)? ?N

DO YOU WISH TO EDIT ANY DATA PAIR (Y/N)? ?Y

WHICH PAIR? ?7

X( 7) = ?7.8
Y( 7) = ?1

DO YOU WISH TO EDIT ANY DATA PAIR (Y/N)? ?N

SORTING DATA...

DO YOU WISH TO SAVE THE CURRENT
DATA ON DISK (Y/N)? ?N

DO YOU WISH TO REMOVE THE
DC COMPONENT (Y/N)? ?N

PLOT DATA (Y/N)? ?Y

WHAT IS THE PLOT WIDTH? ?30

DISPLAY PLOT VALUES (Y/N)? ?Y
```

We are going to repeat the analysis, but make a change in the input data set to test the interpolation procedure.

We can not add more data because the array has been filled. However, if we had answered the original "number of table values" question with 32, and had then load a data file from disk which contained only 16 points, more data could be added at this point.

When a data pair is deleted, the pairs above shift down one to fill the gap. Thus the "current data file" table shown above becomes incorrect above the pair deleted. If data pair 7 is deleted, pair 8 becomes pair 7, pair 9 becomes pair 8, and so on. In this case we deleted the last pair in the table as an example, and we will later replace it.

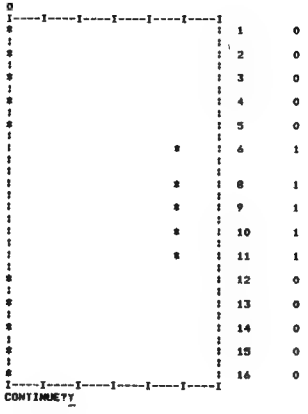
The program just checked the number of data pairs and found only 15. That is an invalid number for entry into the FFT. You will now be given a chance to correct that.

15 points currently exist. The array size initially chosen was 16.
In this case you did not have much choice.

We put the 16^{th} pair back.

Originally pair 7 was (7,1) We will shift that data point to (7.8,1). Note that it is now right next to point 8 which is at (8,1).

The last time we removed the DC component



Point 7 has moved so close to point 8 that it is coincident on the plot. Thus only point 8 is displayed. Note that we are still describing the same square pulse, but this time the points are not evenly spaced.

FOURIER TRANSFORM UNDER WAY....

FREQUENCY CYCLES/UNIT	AMPLITUDE SCALED	PHASE DEGREES
0	1	0
.0625	.7893	191
.125	.308	22
.1875	.1148	34
.25	.2357	225
.3125	.0767	56
.375	.1276	67
.4375	.157	239
.5	0	0

Since we included the DC term this time, we can not make an immediate comparison between the amplitude values calculated for the evenly spaced data and the last set. The amplitude results are always scaled such that the largest value is normalized. However, observe that the phase values are identical.

On to another example.

```
RUN
```

In this example we will see how a data file may be loaded from disk (or cassette) and more data added to it. This particular example is interesting in that the data is not artificial, but rather the result of an experiment. In this experiment a filter was probed with a train of pulses and the output displayed on an oscilloscope. The distorted output represents the "impulse response", and its transform the transfer function.

```
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THE USER INPUTS THE NUMBER
OF DATA POINTS TO BE TREATED.
THIS NUMBER MUST BE AN EVEN POWER
OF TWO. FOR EXAMPLE, N=2^K.
THE PROGRAM WILL THEN PLOT
THE DATA AND ITS SPECTRUM IN
TERMS OF AMPLITUDE AND PHASE.

YOU MAY ALSO CHOOSE TO RECALL
PREVIOUS DATA OR RESULTS.

DO YOU WISH TO PLOT
PREVIOUS RESULTS (Y/N)? ?N


INPUT NUMBER OF TABLE VALUES: ?64

IS THE DATA TO BE LOADED
FROM DISK (Y/N): ?Y

WHAT IS THE FILE NAME: ?RDAR_TA

IS THE DISK READY?Y

 40 POINTS WERE FOUND.

CAUTION! THE FILE CONTAINED
A NUMBER OF DATA PAIRS WHICH IS
NOT AN EVEN POWER OF 2.


CURRENT DATA FILE:

 I     X(I)       Y(I)
---   -------     -------

 1     .00002     0
 2     .00004     .6
 3     .00006     1.5
 4     .00008     2.4
 5     .0001      3
 6     .00012     3.4
 7     .00014     3.6
 8     .00016     3.4
 9     .00018     2.8
 10    .0002      2.2
CONTINUE?Y


 11    .00022     1.5
 12    .00024     .6
 13    .00026     0
 14    .00028     -.6
 15    .0003      -1.2
 16    .00032     -1.6
 17    .00034     -1.8
 18    .00036     -1.9
 19    .00038     -2
 20    .0004      -1.9
CONTINUE?Y
```

The actual set of data consists of 40 data pairs. The next number higher than 40 which will be accepted is 64.

Typo's can usually be corrected before the "RETURN" or "ENTER" key is pressed by using "delete","rubout", "back-arrow", or whatever the corresponding key is on your keyboard.
Only 40 points were found.

The computer quickly becomes aware that the number of current data pairs is not acceptable.

```
 21     .00042     -1.9
 22     .00044     -1.8
 23     .00046     -1.6
 24     .00048     -1.5
 25     .0005      -1.4
 26     .00052     -1.3
 27     .00054     -1.2
 28     .00056     -1.1
 29     .00058     -1
 30     .0006      -.9
CONTINUE?Y

 31     .00062     -.85
 32     .00064     -.8
 33     .001       -.1
 34     .0015      .3
 35     .002       .4
 36     .0025      .35
 37     .003       .25
 38     .0035      .2
 39     .004       .15
 40     .0045      .1
CONTINUE?Y
```

Observe that for long time lengths and low amplitudes, the sampling interval has been increased.

```
DO YOU WISH TO ADD TO DATA (Y/N)! ?Y
```

Two options exist. We can add data to get to 64 pairs, or we can delete data to get to the 32 level. The first option is chosen.

```
YOU MAY ADD  24 POINTS.

HOW MAY POINTS DO YOU WISH TO ADD! ?24

X( 41) = ?.00003
Y( 41) = ?.3

X( 42) = ?.00005
Y( 42) = ?1.06.5

X( 43) = ?.00007
Y( 43) = ?1.95

X( 44) = ?.00009
Y( 44) = ?2.7
```

24 "new" data points are added. They are just the averages of the first 25 points; we are inserting linearly interpolated data.

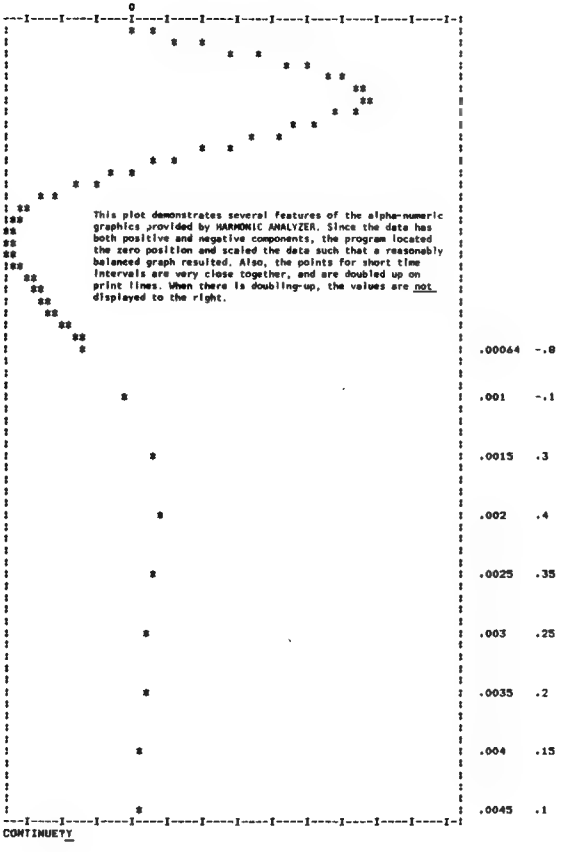
This part of the print-out is boring.

```
DO YOU WISH TO DELETE DATA (Y/N)! ?N

DO YOU WISH TO EDIT ANY DATA PAIR (Y/N)! ?N

SORTING DATA...

DO YOU WISH TO SAVE THE CURRENT
DATA ON DISK (Y/N)! ?Y

WHAT IS THE FILE NAME! ?RDATA

IS THE DISK READY?Y

DO YOU WISH TO REMOVE THE
DC COMPONENT (Y/N)! ?N

PLOT DATA (Y/N)! ?Y

WHAT IS THE PLOT WIDTH! ?65

DISPLAY PLOT VALUES (Y/N)! ?Y
```

The sort takes some time for large data sets. remember that the sort is followed by interpolation to create an equally spaced "data" table.

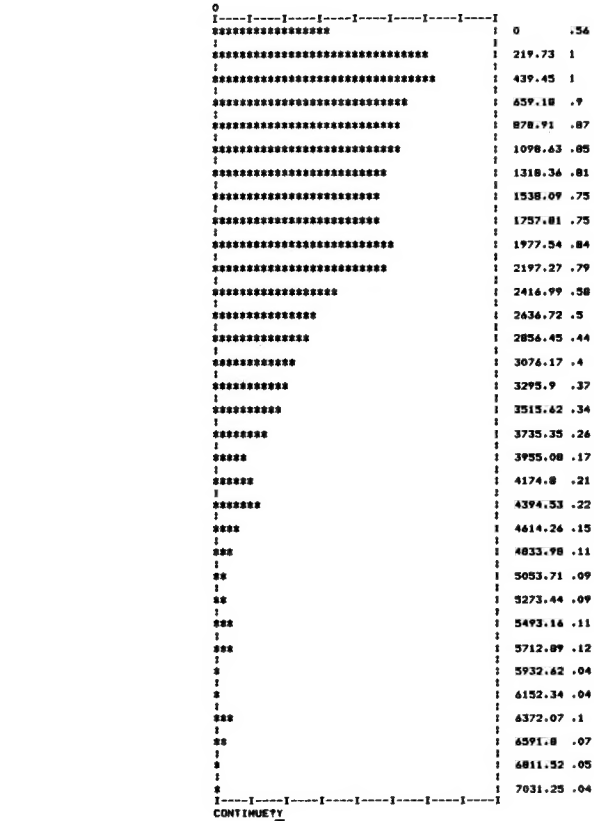
Recall that the file name is always required, but not actually used for the cassette versions.

Always try to use a plot width greater than the number of data pairs.


This plot demonstrates several features of the alpha-numeric graphics provided by HARMONIC ANALYZER. Since the data has both positive and negative components, the program located the zero position and scaled the data such that a reasonably balanced graph resulted. Also, the points for short time intervals are very close together, and are doubled up on print lines. When there is doubling-up, the values are not displayed to the right.
.00064 -.0
.001 -.1
.0015 .3
.002 .4
.0025 .35
.003 .25
.0035 .2
.004 .15
.0045 .1
CONTINUE?Y

```
FOURIER TRANSFORM UNDER WAY....

 FREQUENCY    AMPLITUDE    PHASE
CYCLES/UNIT    SCALED     DEGREES
-----------   ---------   -------

     0           .5404        0
   219.7268      .9953      123
   439.4535      1           44
   659.1802      .9022       10
   878.907       .8696      348
  1098.6338      .849       329
  1318.3606      .8086      312
  1538.0873      .7527      297
  1757.8141      .754       292
  1977.5408      .8377      272
CONTINUE?Y

  2197.2676      .7869      248
  2416.9944      .578       232
  2636.721       .4962      225
  2856.4478      .4439      218
  3076.1746      .4023      209
  3295.9014      .373       200
  3515.628       .3398      188
  3735.3548      .2605      168
  3955.0816      .1748      181
  4174.8082      .2069      188
CONTINUE?Y

  4394.535       .2198      163
  4614.2618      .1533      152
  4833.9885      .1081      152
  5053.7153      .0896      162
  5273.4421      .0907      172
  5493.1688      .1055      170
  5712.8955      .1212      145
  5932.6223      .0419      113
  6152.3491      .0409      222
  6372.0757      .1         182
CONTINUE?Y

  6591.8025      .0726      161
  6811.5293      .0506      161
  7031.2559      .0431      180

DO YOU WISH AN AMPLITUDE
VERSUS FREQUENCY PLOT (Y/N)! ?Y



WHAT IS THE PLOT WIDTH! ?40

DISPLAY PLOT VALUES (Y/N)! ?Y
```

There are 33 transform pairs; a plot width of 40 is sufficient.



This filter actually had no DC response. Most likely there was a small offset in the scope trace from which the data was obtained. Knowing that there was no DC response we could have chosen to arbitrarily remove the DC component before the FFT calculation. That option was not chosen. If it had been, the amplitude would have been zero at zero hertz.

```
DO YOU WISH A PHASE PLOT (Y/N)? Y

WHAT IS THE PLOT WIDTH? 40

DISPLAY PLOT VALUES (Y/N)? Y


0
I----I----I----I----I----I----I----I----I
*                                        :  0       0
:                                        :
###########                              :  219.73  122.61
:                                        :
####                                     :  439.45  44.29
:                                        :
*                                        :  659.18  9.94
:                                        :
##################################       :  878.91  347.55
:                                        :
#################################        :  1098.63 329.48
:                                        :
################################         :  1318.36 311.98
:                                        :
##############################           :  1538.09 297.43
:                                        :
#############################            :  1757.81 292.48
:                                        :
############################             :  1977.54 271.58
:                                        :
########################                 :  2197.27 247.99
:                                        :
#######################                  :  2416.99 231.97
:                                        :
####################                     :  2636.72 225.16
:                                        :
####################                     :  2856.45 217.67
:                                        :
###################                      :  3076.17 209.16
:                                        :
##################                       :  3295.9  200.19
:                                        :
################                         :  3515.62 188.13
:                                        :
###############                          :  3735.35 167.96
:                                        :
################                         :  3955.08 181.09
:                                        :
################                         :  4174.8  188.13
:                                        :
##############                           :  4394.53 162.86
:                                        :
#############                            :  4614.26 151.83
:                                        :
##############                           :  4833.98 152.38
:                                        :
#############                            :  5053.71 142.06
:                                        :
################                         :  5273.44 172.49
:                                        :
################                         :  5493.16 169.59
:                                        :
#############                            :  5712.89 144.58
:                                        :
##########                               :  5932.62 113.36
:                                        :
####################                     :  6152.34 221.93
:                                        :
################                         :  6372.07 182.43
:                                        :
##############                           :  6591.8  161.04
:                                        :
##############                           :  6811.52 160.76
:                                        :
################                         :  7031.25 180
I----I----I----I----I----I----I----I----I

DO YOU WISH TO SAVE THESE RESULTS (Y/N)? Y
                                                    end of examples...
```